**6/6 Day 3:**

**Previously**

Axioms of Probability:

1. Probability must be between 0 and 1.
2. Probability of nothing is nothing
3. The probability of a disjoint to be equal to another is equal to probability.

Probability of everything must add to one.

**Random Variables**

A random variable is a number experiment associated with a random experiment.

Example: Flip a coin 3 times, where x = # of heads

* x is a random variable.
* Support set of x is the set of possible values X may take on. Support set of x = (0,1,2,3).

**Probability Distribution**

The probability x takes on values in its support set.

Example: x = (0,1,2,3)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| k | 0 | 1 | 2 | 3 |
| P(x = k) | 1/8 | 3/8 | 3/8 | 1/8 |

Since P(S) = 1, then the summation of P(X = k) = 1.

**Types of Random Variables**

Two Types of random variables.

1. Discrete: can take on a countable # of values
2. Continuous: X can take on any value in some interval (a, b).

Continuous Random Variables

The Probability Density Function (PDF)

* f(x) such that f(x) >= 0 for all x in R
* the integral of f(x)dx must equal 1, and the area under the curve is the probability.
* P(a < X < b) = Integral of f(x)dx between a and b.

Example: x ~ Exp( lamda = 10)

* “ ~ ” means is distributed
* Has a PDF of if(x > 0 , 1/10(e^(-x/10)) x<0, 0)
* Integrate:
  + Note: take the integral of the function, the P(S) = 1
* Compute Probability:
  + Compute P(x <= 5)
  + Integrate with the boundaries at (0, 5)

The Cumulative Distribution Function (CDF)

* F(t) = Integral (-INF, t) f(x)dx.

Mean and Variance

* Mean = Mu = E[X] = integral xf(x)dx
* Variance = sigma ^2 = integral (-INF, INF) ((x-mu)^2)(f(x)dx)

**6/7 Day 4:**

**Previously**

Look above…

**Computing Mean and Variance**

Using previous equations, computing basic equations. f(x) = lambda(e^(-lambda(x)))

* Mean = mu = (1/lambda)
* Variance = (2/ lambda) – (1/lambda^2) = (1/ lambda^2)
  + Use u-substitution
  + Add a lambda towards the end to make integral look like the mean.

Example: The lifetime of a light bulb, probability density

* f(x) kxe^(-x) ; for x>0 with k as constant.
* What is k?
  + 1 = (k) integral (xe^(-x)dx)
  + Use u-substitution
  + Get [ 0 + integral (0, INF) (e^(-x)dx)
    - This is the equivalent of lambda = 1 and we know that this must be equal to 1.
  + K = 1

Example: Recall r(n) = integral (0, INF) (x^(n-1) \* e^(-x) dx). Rn = (n-1)!

* Mean = 2!
  + Integral (x^(2) \* e^(-x))
  + R(3)
  + 2!
* Variance = 2
  + Integral (x^(3) \* e^(-x) dx) – integrate(mu)
  + R(4) -i(mu)
  + 3! – 2!

Example: F(X); F(X) = integral(0, x) (f(t)dt)

* Can’t see answer; I think it is
* F(X) = 1 – e^(-x) -xe^(-x)

Example: P(5 < x < 10)

* Integrate (5, 10) xe^(-x)
  + Instead use F(X)
  + (1 – e^(-10) – 10e^(-10)) – (1 – e^(-5) – 5e^(-5))
  + e^(-5) + 5e^(-5) – e^(-10) + 10e^(-10)

Example: The density of a package is f(x) = 70/(69x^2) when 1<x<70 else 0

* Verify that f(x) is a density (checking if it integrates to 1)
  + Integrate f(x)dx
  + (70/69)(69/70) = 1

**Homework**

Using example above:

1. Get the CDF
2. Find the chances package weighs 20+ lbs
3. Get mu and sigma^2
4. If Shipping cost $5 per lbs
   1. Get E[Shipping Cost]

**Probability Distribution: Normal Random Variables**

Two parameters are the same, mu and sigma^2

Equation: f(x) = 1 / (sqr(2pi) \* (sigma)) \* (e^((-1/2)((x-mu)/sigma)^2

* This is the normal bell curve.
* Density is symmetric about mu
* The integral = 1
* E[X] = mu
* Variance(x) = sigma^2